

Testing of Dependencies between Stock Returns and Trading Volume by High Frequency Data

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This paper is concerned with a dependence analysis of returns, return volatility and trading volume for five companies listed on the Vienna Stock Exchange and five from the Warsaw Stock Exchange. Taking into account high frequency data for these companies, tests based on a comparison of Bernstein copula densities using the Hellinger distance were conducted. The paper presents some patterns of causal and other relationships between stock returns, realized volatility and expected and unexpected trading volume. There is a linear causality running from realized volatility to expected trading volume, and a lack of nonlinear dependence in the opposite direction. The authors detected strong linear and nonlinear causality from stock returns to expected trading volume. They did not find causality running in the opposite direction. In addition, the existence of fractional cointegration was examined. Despite the equality of the long memory parameters of realized volatility and trading volumes, they do not move together in the long term horizon.

Key Words: realized volatility; trading volume; dynamic interrelations; copulas; fractional cointegration

JEL Classification: G15, C32

Introduction

Market participants usually think that a share price reflects investors' predictions about the future performance of a company. These expectations are based on available information about the firm. The release of new information forces investors to change their expectations about the future performance of the company. New announcements are the main source of price changes. Since investors evaluate the content of new information differently, prices may remain constant even though new information is important for the market. This can be the case if some investors think that the news is good, whereas others understand the same announcements as

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bad news. The direction of movements of prices depends on the average reaction of investors to news.

It is obvious that share prices can be observed if there is a positive trading volume. As with prices, trading volume and changes in it react to the available set of important information on the market. Trading volume reacts in a different way in comparison to stock prices. A change in investors' expectations always leads to a rise in trading volume. The size of trading volume reflects the sum of investors' reactions to news.

A valid response to the question of whether the knowledge of one variable (e. g. volatility) can improve short-run forecasts of other variables is essential for analysts as well as market participants. Thus, in recent years both researchers and investors have focused on the relationship between trading volume, stock returns and return volatility. Most early empirical examinations were concerned with the contemporaneous relationship between price changes and volume.

Both from a theoretical and practical point of view, the dynamic relationship between returns, return volatility and trading volume is much more interesting than the contemporaneous one. One of the most important and useful topics in empirical economics is the examination of the causal relationship between particular variables. The notion of causality was introduced by Granger (1969). It is based on the idea that the past cannot be influenced by the present or future. Thus, if one event is observed before another event, causality can only take place from the first event to the second one.

Many economic and financial time series exhibit the property of long memory. Long-term dependence, called long memory, describes the high-order correlation structure of a time series. If a time series possesses long memory, there is a persistent temporal dependence between observations even when considerably separated in time. The same degree of long memory of two or more time series may indicate some relation between time series. This issue will be addressed in subsequent sections.

The remainder of the paper is set out as follows. A brief review of some aspects of causality and relevant contributions will be given in the next section. The concepts of nonlinear causality and Bernstein copulas are outlined in the third section. A description of the data and the method for estimating volatility are presented in the fourth section.

The empirical results are discussed in the fifth section. Brief conclusions and outlook are given in the last part of the paper.

Literature Review

Karpoff (1987) in his survey of early research about price–volume relations cited important reasons for examining price–volume dependencies. These relations give an insight into the structures of financial markets, and into the information arrival process and how information is disseminated among market participants. This is strictly connected with two hypotheses: the mixture of distributions hypothesis (Clark 1973; Epps and Epps 1976; Tauchen and Pitts 1983; Harris 1986), and the sequential information arrival hypothesis from Copeland (1976) and Tauchen and Pitts (1983).

A knowledge of price–volume relations is useful e. g. in technical analysis and is important with respect to investigations of options and futures markets and in fashioning new contracts.

One of the most often used approaches in research into return–trading volume interrelations is the concept of Granger causality. Causality in the Granger sense can be understood as a kind of conditional dependency.

A serious problem with the linear approach to testing for causality is the low power of tests necessary to detect some kinds of nonlinear causal relations. This problem was raised in contributions, which are concerned with nonlinear causality tests (see e. g. Abhyankar 1998 and Asimakopoulou, Ayling, and Mahmood 2000). The starting point for further investigations was a nonparametric statistical method for uncovering nonlinear causal effects presented by Baek and Brock (1992). In order to detect causal relations the contributors used the correlation integral, an estimator of spatial probabilities across time based upon the closeness of points in hyperspace. The concept of Baek and Brock was improved by Hiemstra and Jones (1994) and Diks and Panchenko (2005; 2006).

The linear and nonlinear causality of companies listed on the DAX index was investigated by Gurgul and Lach (2009). They used daily data at close from January 2001–November 2008. For the testing of nonlinear causality the Diks and Panchenko test was used, while linear dependencies were checked by traditional Vector Autoregressive Models and by a model derived by Lee and Rui (2002). The contributors confirmed the hypothesis that traditional linear causality tests often fail to detect some kinds of nonlinear relations, while nonlinear tests do not. In many cases the test results obtained by the use of empirical data and simulation confirmed a bidirectional causal relationship while linear tests did not detect such causality at all.

Rossi and de Magistris (2010) investigated the relationship between volatility, measured by realized volatility, and trading volume. They showed that trading volume and volatility exhibit long memory but that they are not driven by the same latent factor as suggested by the fractional cointegration analysis. They used fractional cointegration by VAR models as in Nielsen and Shimotsu (2007), and also extended the analysis of Robinson and Yajima (2002) for stationary and nonstationary time series. They found that past (filtered) log-volume has a positive effect on current filtered log-volatility and on current log-volume as well. Their analysis was complemented by using copulas in order to measure the degree of tail dependence.

Bouezmarni, Rombouts, and Taamouti (2012) derived a nonparametric test based on Bernstein copulas and tested using high frequency data for causality between stock returns and trading volume. The contributors proved that at a 5% significance level the nonparametric test clearly rejected the null hypothesis of non-causality from returns to volume, which is in line with the conclusion which followed from the linear test. Further, their nonparametric test also detected a non-linear feedback effect from trading volume to returns at a 5% significance level.

In the next part of this paper in order to check links between the financial variables under study, realized volatility will be used as a proxy for volatility. Our dataset consists of five large companies listed on the WIG20 and five companies listed on the ATX20. The Vienna Stock Exchange is an example of a well-developed small capital market and the Warsaw Stock Exchange represents small emerging markets. The stock markets under study exhibit similar capitalization. Both indexes cover a similar period. ATX index is quoted from 2 January 1991 and the WIG index is used from 16 April 1994.

The Vienna Stock Exchange has been in recent years a local rival of the Warsaw Stock Exchange in Central and Eastern Europe. However recently the stock markets in Vienna and Warsaw have been considering cooperation and in the future a merger of them is possible.

Main Research Conjectures

At the very beginning of our empirical analysis we will check for the stationarity, normality and autocorrelation of the investigated time series. Stationarity is the main assumption of most statistical causality tests, especially of VAR modeling. The literature overview and a preliminary reading of the dataset encouraged us to formulate the following:

CONJECTURE 1 *All time series under study are stationary, non-normal and exhibit an autocorrelation pattern.*

On the basis of previous results derived by the authors for companies listed on the DAX index a linear link between stock returns and expected trading volume may be hypothesized.

In addition, in order to check linear causality a bivariate VAR model is recommended.

CONJECTURE 2 *There is linear causality running from stock returns to expected trading volume for all selected stocks listed on the ATX20 and WIG20.*

However we do not expect a similar interdependence in the case of unexpected trading volume. We predict that:

CONJECTURE 3 *There is no causality between stock returns and unexpected trading volume for stocks selected from ATX20 and WIG20.*

In order to check linear dynamic links for Polish and Austrian stock return volatility and trading volume we will check linear and nonlinear causality between realized volatility and expected (unexpected) trading volume. Taking into account the findings of the other contributors mentioned in the overview we formulate the following conjecture:

CONJECTURE 4 *There is both linear and nonlinear causality running from realized volatility to expected trading volume.*

However, in the light of the economic literature and a preliminary correlation analysis there are no clear linear and nonlinear links between stock return volatility and unexpected trading volume. The correlation analysis does not supply evidence of such interdependencies.

Therefore our next hypothesis is as follows:

CONJECTURE 5 *There is neither linear nor nonlinear causality between realized volatility and unexpected trading volume in either direction.*

In the literature the long memory of financial time series is reported. This property is important in the context of the Mixture of Distribution Hypothesis, which assumes the contemporaneous arrival of random information on the stock market. In particular it is interesting to examine the suggested existence of a latent directing variable which exhibits long memory characteristics and is responsible for the dynamics of realized volatility and volume. The results in previous contributions did not support MDH for daily return volatility and trading volume. Therefore, we also conjecture for the ATX20 and WIG20 that:

CONJECTURE 6 *Realized volatility and trading volume are not fractionally cointegrated. There is no common long-run dependence, and therefore MDH with long memory should be rejected for these stock markets.*

The conjectures listed above will be checked by some recent tests. The details of the testing procedures will be shown in the following sections. The test outcomes depend to some extent on the testing methods applied. After a description of the methodology in the next sections, we will give descriptive statistics of the time series included in our sample.

Nonlinear Causality, Bernstein Copulas and Fractional Cointegration

Now we will present an extension of the Granger causality notion taking into account three variables X , Y and Z . Variable Z is in a causal relation to variable Y , in the Granger sense, if the current values of variable Y can be forecasted more precisely by means of the known past values of variable Z , and those of auxiliary variable X , than in the case where the values of variable Z are not involved in the forecasting process.

In the recent literature on nonlinear dependencies in the sense of Granger causality nonparametric tests are used for the conditional independence of random variables. The conditional independence of random variables implies a lack of causality in the Granger sense. Linton and Gozalo (1997) tested conditional independency by means of a test statistic based on empirical distributions. Su and White (2003) derived a test based on smoothed empirical likelihood functions and in 2007 developed a nonparametric test for the conditional independence of distributions. To this end they applied conditional characteristic functions. The test for conditional independence by Su and White (2008) is based on a kernel estimation of conditional distributions $f(y|x)$ and $f(y|x, z)$. If the null holds true then the last functions are equal. A serious drawback of this test is the restriction of the sum of the dimensions of variables X, Y, Z to seven. In addition, it is necessary to define a weight function for the Hellinger distance necessary to measure the distance between the conditional distributions. The contributors applied their test to examine Granger non-causality in exchange rates. It is used their approach and methodology which is used in the empirical part of this paper. The causality test applied to the detection of nonlinear causality is based on Bernstein copulas (see for example Bouezmarni, Rombouts, and Taamouti 2012).

Bouezmarni, Rombouts, and Taamouti (2012) focused on the differences between their test (henceforth called the BRT test) and the test by Su and White (2008). The main differences and advantages of the BRT test can be summarized as follows:

1. There is no restriction on the sum of the dimensions of the variables under study.
2. The application of nonparametric Bernstein copulas in order to estimate the joint conditional distributions guarantees the non-negativity of their distributions. This is important with respect to a true determination of the distance between them by means of Hellinger distance.
3. It is necessary to determine only one parameter, which determines the accuracy of the estimation of nonparametric copula density.

The contributors demonstrated by means of simulation studies that their test has appropriate power and facilitates the recognition of different nonlinear dependencies between variables. By means of simulation exercises evidence is presented for the uselessness of a classic linear causality test for the detection of causal dependencies between nonlinear processes. The authors applied their test in a Granger non-causality examination of many macroeconomic and financial variables.

**NONLINEAR CAUSALITY VERSUS
CONDITIONAL DEPENDENCE**

Let $\{(X'_t, Y'_t, Z'_t) \in \mathbb{R}^{d_1} \times \mathbb{R}^{d_2} \times \mathbb{R}^{d_3}, t = 1 \dots T\}$ be the realization of the stochastic process in \mathbb{R}^d , where $d = d_1 + d_2 + d_3$ with joint distribution F_{XYZ} and density function f_{XYZ} . The test of conditional independency between variables Y and Z under condition X can be written down for density functions as (Bouezmarni, Rombouts, and Taamouti 2012):

$$H_0: P(f_{(Y|X,Z)}(y|x, z) = f_{(Y|X)}(y|x)) = 1, \quad \text{for } \forall y \in \mathbb{R}^{d_2} \quad (1)$$

$$H_1: P(f_{(Y|X,Z)}(y|x, z) = f_{(Y|X)}(y|x)) < 1, \quad \text{for some } y \in \mathbb{R}^{d_2} \quad (2)$$

where $f_{(\cdot|\cdot)}(\cdot|\cdot)$ stands for the conditional density function.

It is worth noting that a lack of causality in the Granger sense can be understood as conditional independence. Let $(Y, Z)'$ be a Markov process of order 1. The variable Z does not cause in the Granger sense variable Y if and only if the following null hypothesis holds true:

$$H_0: P(f_{(Y|X,Z)}(y_t|y_{(t-1)}, z_{(t-1)}) = f_{(Y|X)}(y_t|y_{(t-1)})) = 1,$$

i. e. $y = y_t, x = y_{(t-1)}, z = z_{(t-1)}$ for $d_1 = d_2 = d_3 = 1$.

For the sake of simplicity of notation we assume $d_i = 1$ for $i = 1, 2, 3$. Taking into account this notation the well-known Sklar theorem can be put down in the form:

$$F_{XYZ}(x, y, z) = C_{XYZ}(F_X(x), F_Y(y), F_Z(z)).$$

The respective density function f_{XYZ} is given by the equation

$$f_{XYZ}(x, y, z) = f_X(x)f_Y(y)f_Z(z)c_{XYZ}(F_X(x), F_Y(y), F_Z(z)),$$

where c_{XYZ} is the density function of copula C_{XYZ} . The null hypothesis (1) can be expressed by means of the copula notion in the following form:

$$\begin{aligned} H_0: & P(c_{XYZ}(F_X(x), F_Y(y), F_Z(z))) \\ & = c_{XY}(F_X(x), F_Y(y))c_{XZ}(F_X(x), F_Z(z))) = 1, \quad \forall y \in \mathbb{R} \end{aligned}$$

while an alternative hypothesis fulfills the inequality:

$$\begin{aligned} H_1: & P(c_{XYZ}(F_X(x), F_Y(y), F_Z(z))) \\ & = c_{XY}(F_X(x), F_Y(y))c_{XZ}(F_X(x), F_Z(z))) < 1 \end{aligned}$$

for some $y \in \mathbb{R}$, where c_{XY} and c_{XZ} stand for the densities of the copulas of two dimensional distributions (X, Y) and (X, Z) . The test statistics suggested by Bouezmarni, Rombouts, and Taamouti (2012) is based on the Hellinger distance between two distributions i. e. the density of the copula c_{XYZ} and the product of the densities of copulas c_{XY} and c_{XZ} . This measure

$$H(c, C) = \int_{[0,1]^3} \left(1 - \sqrt{\frac{c_{XY}(u, v)c_{XZ}(u, w)}{c_{XYZ}(F_X(x), F_Y(y), F_Z(z))}} \right)^2 dC_{XYZ}(u, v, w) \quad (3)$$

is equal to 0 if the null hypothesis holds true.

The distance (3) exhibits important advantages. First of all it is symmetric and invariant with respect to monotone transformations. In addition, it is not sensitive to outliers, because their weights are lower than the weights of other observations. For empirical data Hellinger distance (3) can be estimated by means of the following formula:

$$\begin{aligned} \hat{H} &= H(\hat{c}, C_T) = \int_{[0,1]^3} \left(1 - \sqrt{\frac{\hat{c}_{XY}(u, v)\hat{c}_{XZ}(u, w)}{\hat{c}_{XYZ}(u, v, w)}} \right)^2 dC_{XYZ}(u, v, w) \\ &= \frac{1}{T} \sum_{i=1}^T \left(1 - \sqrt{\frac{\hat{c}_{XY}(\hat{F}_X(X_T), \hat{F}_Y(Y_T))\hat{c}_{XZ}(\hat{F}_X(X_T), \hat{F}_Z(Z_T))}{\hat{c}_{XYZ}(\hat{F}_X(X_T), \hat{F}_Y(Y_T), \hat{F}_Z(Z_T))}} \right)^2, \end{aligned}$$

where $\hat{F}(\cdot)$ is the empirical form of marginal distribution $F(\cdot)$. In addition, the densities of copulas are estimated by means of nonparametric methods.

The test statistics (called *BRT*) and the method for computing the *p*-value is given in Bouezmarni, Rombouts, and Taamouti (2012).

$$BRT = \frac{Tk^{-\frac{3}{2}}}{\sigma} (4\hat{H} - C_1 T^{-1} k^{\frac{3}{2}} - \hat{B}_1 T^{-1} k - \hat{B}_2 T^{-1} k - \hat{B}_3 T^{-1} k^{\frac{1}{2}}),$$

where $C_1 = 2^{-3}\pi^{\frac{3}{2}}$, $\sigma = \sqrt{2(\frac{\pi}{4})^{\frac{3}{2}}}$ and

$$\hat{B}_1 = -2^{-1}\pi + T^{-1} \sum_{t=1}^T \frac{(4\pi\hat{G}_{t1}(1 - \hat{G}_{t1}))^{-\frac{1}{2}}(4\pi\hat{G}_{t2}(1 - \hat{G}_{t2}))^{-\frac{1}{2}}}{\hat{c}_{XY}(G_{t1}, G_{t2})},$$

$$\hat{B}_2 = -2^{-1}\pi + T^{-1} \sum_{t=1}^T \frac{(4\pi\hat{G}_{t1}(1 - \hat{G}_{t1}))^{-\frac{1}{2}}(4\pi\hat{G}_{t3}(1 - \hat{G}_{t3}))^{-\frac{1}{2}}}{\hat{c}_{XY}(G_{t1}, G_{t3})},$$

$$\hat{B}_3 = \pi^{-\frac{1}{2}} T^{-1} \sum_{t=1}^T \frac{1}{\sqrt{\hat{G}_{t1}(1 - \hat{G}_{t1})}}.$$

The densities \hat{c}_{XYZ} , \hat{c}_{XY} and \hat{c}_{XZ} are estimated by means of Bernstein copulas. Under the null hypothesis the test statistics is distributed asymptotically according to standard normal distribution. The null hypothesis is rejected for a given significance level α if $BRT > z_\alpha$ holds true, where z_α denotes the critical value given in the tables of standard normal distribution. Taking into account that the test statistic is asymptotically normal, the contributors advise in the case of a finite sample the calculation of *p*-values by means of bootstrap methods. Classical bootstrap methods referring to empirical distribution cannot be applied. That is why Paparoditis and Politis (2000) suggested a local bootstrap method for nonparametric kernel estimators. They take into account the fact that the densities of the variables are conditional. This method was applied by Bouezmarni, Rombouts, and Taamouti (2012) and Su and White (2008). The *p*-values can be determined for samples $\{X_t^*, Y_t^*, Z_t^*\}_{t=1}^T$ generated by bootstrapping under condition $d_1 = d_2 = d_3 = 1$ in the following steps:

1. In the first step X_t^* is generated by means of a kernel estimator:

$$\tilde{f}(x) = T^{-1}h^{-1} \sum_{t=1}^T L\left(\frac{X_t - x}{h}\right),$$

where L stands for the density of the one dimensional distribution.

For $t = 1, \dots, T$ the values of Y_t^* and Z_t^* should be generated independently from conditional densities:

$$\tilde{f}(y|X_t^*) = \frac{\sum_{s=1}^T L\left(\frac{Y_s - y}{h}\right) L\left(\frac{X_s - X_t^*}{h}\right)}{\sum_{s=1}^T L\left(\frac{X_t - X_s^*}{h}\right)},$$

$$\hat{f}(z|X_t^*) = \frac{\sum_{s=1}^T L\left(\frac{Z_s - z}{h}\right) L\left(\frac{X_s - X_t^*}{h}\right)}{\sum_{s=1}^T L\left(\frac{X_t - X_s^*}{h}\right)}.$$

2. For the generated sample test statistic BRT^* should be established.
3. Steps 1–3 should be repeated M times in order to receive $\{BRT_j^*\}_{j=1}^M$.
4. Finally, the bootstrap p -value is given by

$$p^* = \frac{1}{M} \sum_{j=1}^M 1_{\{BRT_j^* > BRT\}}.$$

FRACTIONAL COINTEGRATION

For any d and d_e two $I(d)$ processes are fractionally cointegrated if there exists a linear combination that is $I(d_e)$ with $d_e < d$. In this case there exists long-run dependence and a common stochastic trend. Assume that $z_t = (x_t, y_t)$ with $x_t \in I(d)$ and $y_t \in I(d)$. If there exists $\beta \neq 0$ such that the linear combination $y_t - \beta x_t \in I(d_e)$, $0 \leq d_e < d$, then x_t and y_t are fractionally cointegrated. We write $z_t \in CI(d, b)$, for $b = d - d_e$. Robinson and Yajima (2002) consider the case of stationary variables, while Nielsen and Shimotsu (2007) also analyse the case of covariance nonstationary variables.

The fractional cointegration can be tested as follows. Firstly, using Whittle estimators the long memory parameters are estimated, and then a test of their equality is performed (comp. Robinson and Yajima 2002).

Data Description and Estimation of Realized Volatility

We consider two original datasets. Firstly there are five-minute transaction prices and volumes of five stocks from the Warsaw Stock Exchange from 3 March 2008 to 28 January 2011 (732 daily observations). The selected stocks are BRE Bank SA (BRE), BZ WBK SA (BZW), KGHM Polska Miedź SA (KGH), Bank Polska Kasa Opieki SA (PEO), Polskie Górnictwo Naftowe i Gazownictwo SA (PGN). The second sample contains the tick-by-tick transaction prices and volumes of five stocks from the Vienna Stock Exchange from 2 January 2009 to 9 November 2011 (711 daily observations). The selected stocks are Andritz AG (ANDR), Erste

Group Bank AG (EBS), OMV AG (OMV), Telekom Austria AG (TKA) and Voestalpine AG (VOE). For these companies' descriptive statistics of the time series of returns, realized volatility and trading volume were computed. They are presented below.

DAILY STOCK RETURNS

We computed daily stock returns at close and multiplied them by 100. The descriptive statistics and tests conducted confirmed stylized facts about stock returns r_t . The departure from normality is reflected in kurtosis and skewness. The null hypothesis about normality is rejected for all companies under study (Jarque-Bera test). The Ljung-Box test indicates that in most cases there exists significant autocorrelation in stock returns.

REALIZED VOLATILITY

In empirical investigations daily squared returns or absolute returns are used as a proxy of volatility. For high frequency, realized volatility is the better alternative, because of improving the accuracy of risk computed with high frequency squared returns.

In this paper we use a Newey-West estimator based on a Bartlett kernel for daily-realized volatility (Hansen and Lunde, 2005):

$$RV_t^{NW} = \sum_{i=1}^m r_{i,t}^2 + 2 \sum_{k=1}^q \left(1 - \frac{k}{q+1}\right) \sum_{i=1}^{m-k} r_{i,t} r_{i+k,t}.$$

This estimator has many advantages. However, it does not take into account volatility in the time between closing the session and opening the session next day. Therefore, it is necessary to add to RV_t^{NW} a square of return computed for the price at close and price at open denoted by r_{COt} . We followed a procedure by Hansen and Lunde (2005). In addition, for companies listed on the Vienna Stock Exchange an optimal frequency parameter was estimated.

We applied a logarithmic transformation to the realized volatility series. We observe that in spite of this logarithmic transformation, almost all time series are not normally distributed (the exceptions being *BRE* and *PEO*). Significant autocorrelation is observed for all stocks under study. We observe that all of the series from the Vienna Stock Exchange are positively skewed. Next, we removed the deterministic trend from the time series. The series adjusted in this way are denoted as $\ln RV_t$. In all cases the null hypothesis of unit root is rejected, so the series $\ln RV_t$ can be used in VAR models.

TRADING VOLUME

Daily trading volume is computed as the sum of volumes corresponding to each transaction from a whole given day. We compute the descriptive statistics of the log-volume series. In the case of the Polish stock PGN and the Austrian stocks EBS, TKA and VOE the null hypothesis of normality is rejected. We filtered the log-volume from the deterministic trend and calendar effects.

In the next sections we consider two types of trading volume: expected and unexpected. Unexpected trading volume ($\widetilde{\ln V}_t$) is that part of total volume that cannot be forecasted and is generated by the random process of new pieces of information coming to the market. Expected trading volume ($\ln V_t$) can be forecasted and we used fitted values of ARMA models as a proxy. Unexpected trading volume is given by the residuals from ARMA models. Taking into consideration that in the next sections VAR models are used, we conducted an augmented Dickey-Fuller test for unit root for the variables under study. To summarize, the properties of the time series under study are in line with Conjecture 1.

Empirical Results and Their Analysis

CAUSALITY

In this section we analyse pairwise by means of nonparametric Bernstein copulas nonlinear causality between prices, trading volume and realized volatility.

Causal Price-Trading Volume Relations

To test linear Granger causality we applied bivariate VAR(k). In order to test nonlinear causality we used the BRT statistics described in the previous section applied to residuals from VAR models. Using such a method we can be sure that we test only nonlinear relations. When estimating Bernstein copulas we took bandwidth k as integer part of $2\sqrt{T}$. We computed the p -values of the test with 200 bootstrap samples. Below we used the notations $X \rightarrow Y$ in order to describe the null hypothesis: that X does not Granger cause Y .

Stock Returns and Expected Trading Volume

The hypotheses

$$H_0: r_t \not\rightarrow \ln V_t,$$

$$H_1: r_t \rightarrow \ln V_t$$

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TABLE 1 Results of causality testing for stock returns and expected trading volume

WIG20					ATX20				
H_0	$r_t \rightarrow \ln V_t$		$\ln V_t \rightarrow r_t$		H_0	$r_t \rightarrow \ln V_t$		$\ln V_t \rightarrow r_t$	
Test	Linear	BRT	Linear	BRT	Test	Linear	BRT	Linear	BRT
BRE	0.116	0.000	0.588	0.225	ANDR	0.000	0.000	0.822	0.110
BZW	0.125	0.035	0.363	0.355	EBS	0.070	0.000	0.702	0.110
KGH	0.796	0.005	0.754	0.145	OMV	0.064	0.000	0.291	0.180
PEO	0.029	0.005	0.003	0.355	TKA	0.001	0.010	0.837	0.055
PGN	0.159	0.000	0.146	0.225	VOE	0.004	0.000	0.103	0.605

in terms of conditional densities can be formulated as follows:

$$H_0: f(\ln V_t | \ln V_{t-1}, r_{t-1}) = f(\ln V_t | \ln V_{t-1}),$$

$$H_1: f(\ln V_t | \ln V_{t-1}, r_{t-1}) \neq f(\ln V_t | \ln V_{t-1}).$$

The opposite direction of causal dependency has the form:

$$H_0: f(r_t | r_{t-1}, \ln V_{t-1}) = f(r_t | r_{t-1}),$$

$$H_1: f(r_t | r_{t-1}, \ln V_{t-1}) \neq f(r_t | r_{t-1}).$$

Table 1 presents the p -values of the tests conducted.

Bidirectional linear causality was detected only for one Polish stock (PEO). On the other hand there is linear causality from stock returns to trading volume for all stocks from ATX, but not in the opposite direction. The results concerning nonlinear dependencies showing that stock returns cause expected trading volume are the same for both sets of stocks under study. The null hypothesis of lack of causality is rejected. With one exception (TKA) causality in the opposite direction is not detected. The computation results mean that Conjecture 2 holds true.

Stock Returns and Unexpected Trading Volume

Firstly, we estimated a bivariate VAR model for pair $r_t - \widetilde{\ln V}_t$. As in previous sections we used an empirical distribution function in order to transform the residuals from this model. The respective hypotheses are:

$$H_0: r_t \rightarrow \widetilde{\ln V}_t \text{ against } H_1: r_t \rightarrow \ln \widetilde{V}_t, \text{ and}$$

$$H_0: \widetilde{\ln V}_t \rightarrow r_t \text{ against } H_1: \ln \widetilde{V}_t \rightarrow r_t.$$

There is no nonlinear relationship in either direction. Linear causality from returns to unexpected trading volume was detected for BRE, PEO

TABLE 2 Results of causality testing for stock returns and unexpected trading volume

WIG20					ATX20				
H_0	$r_t \rightarrow \ln \bar{V}_t$		$\ln \bar{V}_t \rightarrow r_t$		H_0	$r_t \rightarrow \ln \bar{V}_t$		$\ln \bar{V}_t \rightarrow r_t$	
Test	Linear	BRT	Linear	BRT	Test	Linear	BRT	Linear	BRT
BRE	0.030	0.565	0.679	0.740	ANDR	0.031	0.100	0.042	0.540
BZW	0.331	0.930	0.076	0.680	EBS	0.107	0.155	0.508	0.800
KGH	0.753	0.690	0.200	1.000	OMV	0.411	0.705	0.051	0.415
PEO	0.057	0.415	0.006	0.765	TKA	0.227	0.880	0.669	0.965
PGN	0.425	1.000	0.974	0.505	VOE	0.114	0.980	0.869	0.750

TABLE 3 Results of testing for the pair realized volatility – expected trading volume

WIG20					ATX20				
H_0	$\ln RV_t \rightarrow \ln V_t$		$\ln V_t \rightarrow \ln RV_t$		H_0	$\ln RV_t \rightarrow \ln V_t$		$\ln V_t \rightarrow \ln RV_t$	
Test	Linear	BRT	Linear	BRT	Test	Linear	BRT	Linear	BRT
BRE	0.000	0.840	0.029	0.280	ANDR	0.000	0.675	0.823	0.690
BZW	0.001	0.230	0.457	0.785	EBS	0.000	0.155	0.002	0.110
KGH	0.000	0.200	0.159	0.820	OMV	0.000	0.365	0.766	0.380
PEO	0.000	0.045	0.781	0.675	TKA	0.000	0.005	0.005	0.190
PGN	0.000	0.225	0.022	0.450	VOE	0.000	0.225	0.022	0.690

and ANDR. When considering causality from unexpected trading volume to returns we reject the null hypothesis for two Polish stocks (BZW and PEO), and two Austrian ones (ANDR and OMV). In the light of these results Conjecture 3 is true only to some extent.

Realized Volatility and Expected Trading Volume

The linear, causal relations between realized volatility and expected trading volume were tested with the VAR model described above. To test the presence of nonlinear relations we formulated the following null hypotheses

$$H_0: f(\ln RV_t | \ln RV_{t-1}, \ln V_{t-1}) = f(\ln RV_t | \ln RV_{t-1}) \quad \text{and}$$

$$H_0: f(\ln V_t | \ln V_{t-1}, \ln RV_{t-1}) = f(\ln V_t | \ln V_{t-1}).$$

The first of these hypotheses is equivalent to $H_0: \ln V_t \rightarrow \ln RV_t$ and the second to $H_0: \ln RV_t \rightarrow \ln V_t$. Table 3 summarizes the results of testing (p -values).

In all cases there is linear causality running from realized volatility to

TABLE 4 Results of testing for the pair realized volatility – unexpected trading volume

WIG20					ATX20				
H_0	$\ln RV_t \rightarrow \widetilde{\ln V}_t$		$\widetilde{\ln V}_t \rightarrow \ln RV_t$		H_0	$\ln RV_t \rightarrow \widetilde{\ln V}_t$		$\widetilde{\ln V}_t \rightarrow \ln RV_t$	
Test	Linear	BRT	Linear	BRT	Test	Linear	BRT	Linear	BRT
BRE	0.897	0.705	0.207	0.680	ANDR	0.750	0.990	0.049	0.995
BZW	0.112	0.265	0.319	0.230	EBS	0.039	0.705	0.002	0.965
KGH	0.189	0.900	0.875	0.965	OMV	0.149	1.000	0.325	0.605
PEO	0.035	0.735	0.682	0.525	TKA	0.141	0.890	0.254	0.990
PGN	0.253	0.785	0.008	0.910	VOE	0.016	0.995	0.013	0.915

expected trading volume. Causality in the opposite direction is detected only in the case of BRE, PGN, EBS, TKA and VOE. In addition, there is nonlinear causality from $\ln RV_t$ to $\ln V_t$ for one Polish stock (PEO) and one Austrian company (TKA). Nonlinear dependencies in the opposite direction were not detected. The results of computations partly support Conjecture 4.

Realized Volatility and Unexpected Trading Volume

We now replace $\ln V_t$ with $\widetilde{\ln V}_t$ and estimate the VAR models and the required copulas again. The hypotheses under study are the following:

$$\begin{aligned}
 H_0: \ln RV_t \rightarrow \widetilde{\ln V}_t \quad \text{against} \quad H_1: \ln RV_t \rightarrow \widetilde{\ln V}_t \quad \text{and} \\
 H_0: \widetilde{\ln V}_t \rightarrow \ln RV_t \quad \text{against} \quad H_1: \widetilde{\ln V}_t \mapsto \ln RV_t
 \end{aligned}$$

In all cases there is no nonlinear causal relationship in either direction, which is in line with Conjecture 5. We observed linear causality from realized volatility to unexpected trading volume in three cases (PEO, EBS and VOE). Causality in the opposite direction is detected for one Polish stock PGN and three stocks from ATX (ANDR, EBS, VOE). These findings contradict Conjecture 5.

LONG MEMORY ESTIMATION RESULTS

We use the Whittle estimation method and perform a test for the equality of long memory parameters. The functions $h(n)$ as in (Robinson and Yajima 2002) are:

$$\begin{aligned}
 h_1(n) &= \frac{1}{\ln n}, \\
 h_2(n) &= \frac{1}{\ln^2 n}.
 \end{aligned}$$

TABLE 5 Results of long memory parameter estimation and equality testing

WIG20					ATX20				
Test	$d_{\ln RV_t}$	$d_{\ln V_t}$	$T_o(h_1)$	$T_o(h_2)$	Test	$d_{\ln RV_t}$	$d_{\ln V_t}$	$T_o(h_1)$	$T_o(h_2)$
BRE	0.271	0.393	1.274	1.616	ANDR	0.397	0.255	2.143	2.904
BZW	0.366	0.305	0.297	0.371	EBS	0.465	0.316	2.436	3.373
KGH	0.425	0.370	0.297	0.390	OMV	0.329	0.248	0.638	0.846
PEO	0.413	0.263	1.930	2.446	TKA	0.386	0.152	5.151	6.767
PGN	0.423	0.303	1.169	1.461	VOE	0.495	0.382	1.630	2.407

The parameter $m = n^{0.6}$ is equal to 52 for Polish stocks and to 51 for Austrian stocks. The standard errors of the estimation of long memory parameters are 0.136 and 0.137, respectively. The table 5 presents the results of estimation and testing

The results presented above are in line with the results of unit root testing. All of the series are stationary and exhibit long memory. With one exception (TKA) all of the estimated parameters are significant at a 0.1 level (most of them are significant at a 0.05 level and below). Taking into account that the values of the chi-square distribution with one degree of freedom are equal to $\chi_1^2 = 2.706$, $\chi_1^2 = 3.841$, $\chi_1^2 = 6.635$ for significance levels 0.1, 0.05 and 0.01 respectively, there is no reason to reject the null hypothesis for any Polish stock under consideration. In the case of the Austrian stocks we reject the null for the TKA stock (when using both h_1 and h_2 functions). When considering only the h_2 function we also reject the null in the cases of ANDR and EBS. In table 5 the results of the estimated rank of cointegration are presented (we omit the TKA stock here). The parameter $m_1 = n^{0.55}$ used in the estimation is equal to 37 for both sets of stocks and we multiply the eigenvalues by 10000. The estimation results (details of them are available from the authors upon request) indicate that in all cases of stocks traded on the Warsaw Stock Exchange the rank of cointegration is equal to 0. This means that despite the equality of the long memory parameters, a linear combination with a lower degree of integration doesn't exist. In the case of the EBS and VOE stocks there exists one cointegrating vector, but only for $v(n) = m_1^{-0.25}$. To summarize, the results reported here are in line with Conjecture 6.

Conclusions

The main criterion for the maturity of a financial market are properties of the information flow process such as degree of asymmetry, the speed

at which new information is reflected in prices and trading volume, the strength and types of short-and long-term, linear and non-linear, contemporaneous and causal relationships between the different characteristics of shares (e. g. price, returns, return volatility, trading volume).

The main goal of this research was to check by using high frequency data the causal price-volume relationships for selected highly liquid stocks traded on the Warsaw and Vienna stock exchanges and compare by this means these stock markets. The knowledge of these relations allows getting an insight into the structure of both capital markets. The main question was: how is information disseminated among market participants?

The reply to this question is strictly connected with two central and contradictory research hypotheses: contemporaneous information arrival (the mixture of distributions hypothesis by Clark) or the sequential information arrival hypothesis (formulated by Copeland). The proper tool to check this research problem is modern causality analysis.

To detect linear causality classical vector autoregressive models were used. The nonlinear form of relationships was examined using a test based on nonparametric copulas.

In order to check the conditional dependence between two vector processes the authors applied a new test defined by Bouezmarni, Rombouts, and Taamouti (2012). This test is based on nonparametric estimation and Bernstein copulas. The common test statistics require an estimation of copula density functions. The nonparametric estimator of copula density is based on Bernstein polynomials. The Bernstein copula estimator is always non-negative and does not suffer from the boundary bias problem. This test is time consuming but easy to conduct. The main reason for this is that it does not involve a weighting function in the test statistic. In addition, it can be applied in general settings since there is no restriction on the dimension of the data. To apply this test, only a bandwidth for the nonparametric copula is needed.

The volatility of stock returns was computed using realized volatility estimators including changes in prices for non-trading hours. There are some clear patterns of causal relationships between stock returns, realized volatility and expected and unexpected trading volume.

The conjecture about stationarity was by and large supported by empirical results concerning returns, return volatility and trading volume for companies under study from both stock markets.

As regards the pair stock returns and trading volume, conclusions de-

pend on the part of trading volume used. There is strong linear and non-linear causality from stock returns to expected trading volume, and a lack of such a relationship in the opposite direction. So knowledge of past stock returns can improve forecasts of expected trading volume. When comes to unexpected trading volume, we conclude that there is only a linear, causal relationship from stock returns to unexpected trading volume. Neither linear nor nonlinear causal relations in the opposite direction (from trading volume to returns) are detected. The empirical results imply that, although there is a positive contemporaneous correlation between trading volume and returns, trading volume does not add significant predictive power in the forecast of future returns in the presence of current and past returns. This finding is consistent (for both markets under study) with the Clark (1973) mixture model, which predicts no causal relation from trading volume to stock returns. The empirical results also underlined the difficulty of improving the predictability of returns by adding public information about trading volume.

There is a linear causality running from realized volatility to expected trading volume, and a lack of nonlinear dependence in the opposite direction. When unexpected trading volume is used, we observe (with one exception) linear causality for the pair volatility and trading volume in both directions and a lack of nonlinear causality.

The results reported above mean that trading volume helps to predict return volatility and vice versa. Trading volume helps to predict return volatility. However, it is unable to forecast the level of returns. In other words, trading volume contains information about returns indirectly through its predictability of return volatility. This finding supports the Clark (1973) latent common-factor model. In this model trading volume is defined as a proxy for daily information flow in the stochastic process generating variance of stock returns.

The authors also investigated the properties of realized volatilities and trading volumes series with respect to long memory. The series under study (filtered realized volatilities and trading volumes) exhibit long memory and in most cases degrees of fractional integration are equal (especially for stocks listed on the Warsaw Stock Exchange), which means that they share common long-run dependence. This evidence supports a modified version of the mixture-of-distribution hypothesis of Bollerslev and Jubinski (1999), which posits the existence of a latent directing variable possessing long memory characteristics which account for the dynamics of volatility and trading volume. Our results reflecting the infor-

mation arrival process confirmed the existence of long memory. This may allow us to generalize models that are based only on short-run dynamics and can help to provide a better characterization of joint volatility-trading volume dependencies. In addition, the existence of fractional cointegration was examined. Despite the equality of the long memory parameters of realized volatilities and trading volumes, by and large linear combinations of these variables with a lower degree of fractional integration do not exist, so they do not move together in the long time horizon. In other words a mutual long-run dependence does not exist. These findings are in general not consistent with an MDH with long memory.

To summarize, the relationships returns-return volatility-trading volume are similar for samples of companies listed on both stock markets under study. The findings based on high frequency data are in favour of the MDH hypothesis. However the empirical results did not support MDH with long memory. The findings mean that it is hard to forecast returns based on past values of trading volume.

One of the main limitations of this analysis is the unavailability and high cost of high frequency data. Future analyses for comparing the actual degree of development of the WSE with the stock exchange in Vienna should be performed in subperiods of bear and bull markets on the basis of intraday data for all companies listed on both indices under study. In this way the stability and robustness of results and interdependencies could be checked. In addition, on the basis of high frequency data the basic characteristics of the microstructure of these stock markets should be examined and compared.

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