Regime-Dependent Relationships among Stock Markets in Frankfurt, Vienna and Warsaw

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This paper analyzes short-run relationships between German, Austrian and Polish stock market indices using the Markov Switching VAR (MSVAR) model. The impulse response function is used as the main tool and reveals two market phases. The results are useful for investors; reactions to disturbances are complex and depend on the market phase and the considered pair of variables. There is also a theoretical analysis of the MSVAR model. The theoretical unconditional characteristics of the process driven by the MSVAR model are presented along with calculation techniques which can be applied to other models.

**Key Words:** Markov switching VAR, regime-dependent impulse response, stock markets, dynamic relationship

**JEL Classification:** F36, G15

**Introduction**

The interdependencies between international stock markets have been investigated in many papers. The financial crises which arose in October 1987 and September 2008 provide convincing evidence of the increasing interdependence of global stock markets. A knowledge of the mutual links between different stock markets is important for both investors and policy makers. The hedging and diversification strategies being used by market participants are closely tied to the nature and strength of these interrelationships. If two stock markets are strongly interrelated, it implies that a hedging strategy will not be simple and diversification opportunities will be considerably reduced. Additionally, links between stock markets have an effect on policy makers. They make regulatory policy more complex on the domestic market, because the shocks which come from global stock markets may have an impact on the domestic market, and in consequence a crisis can be ‘imported.’ Taking into account the importance of possible links, economists have focused on interdependencies.
between particular stock markets. An important research problem is the question of how shocks spread to other stock markets (the contagion effect). Researchers have attempted to gain an insight into these processes and determine the factors which motivate these interactions. A knowledge of these mechanisms will provide better understanding of the contagion, hedging and diversification effects observed especially in times of financial crisis.

Underreaction to good news and overreaction to bad news are widely observed on financial markets and can imply that in times of crisis (the bear phase of a stock market) stock returns tend to be more dependent on each other than in optimistic times (the bull phase of a stock market). As regards the risk level of an international portfolio, investors in a bear market phase may lose the advantage of diversification i.e. international portfolios may be more risky than market participants suspect. This asymmetric interdependence is a source of rising diversification costs when it comes to foreign stocks.

The goal of this article is to examine the short-run interrelationships between the stock markets of Germany (the Frankfurt Stock Exchange, FSE), Austria (the Vienna Stock Exchange, VSE) and Poland (the Warsaw Stock Exchange, WSE) represented by their main indices: the DAX30, the ATX20 and the WIG20, respectively. The relationships between these three stock markets are of interest for the following reasons. Firstly, since Germany is the largest economy in the European Union, fluctuations in the real German economy have a significant impact on both Austria and Poland. Secondly, Germany is the prime trading partner of both Austria and Poland. Thirdly, the Vienna Stock Market represented by the ATX20 index is, for cultural, historical, geographical and economical reasons, closely associated with the German Stock Exchange. Moreover, the Vienna Stock Exchange is a local rival of the Warsaw Stock Exchange in Central and Eastern Europe. The Warsaw Stock Exchange represents an emerging stock market of an economy in transition, while the Frankfurt and Vienna stock markets represent well developed markets of different sizes. The VSE exhibits much lower capitalization than the FSE. The WSE and the VSE are similar in terms of capitalization. Taking into account the above reasons we expect that these stock markets are closely linked.

Contributions to date have not been concerned with possible structural breaks between bull and bear regimes in these stock markets. Further, many studies have provided evidence, that stock return characteristics and dynamic links between stock markets vary considerably during
bull and bear market phases. Ignoring these structural breaks may lead to incomplete or incorrect statistical results.

We have chosen the Markov switching vector autoregressive (MSVAR) model to describe the interdependence between these markets under study. We provide a complex analytical characterization of the unconditional distribution of the variable driven by the MSVAR model. We interpret the results and explain how parameter changes affect these characteristics. A convenient and illustrative method for presenting the strength and dynamics of an interrelation is to calculate the impulse response function (IRF). In the case of regime switching models, the regime-dependent IRF is used. We present two different approaches to the regime-dependent IRF. In our opinion this approach provides new insights into the issue of dynamic relationships between stock markets.

Especially we try to compare the strength and dynamics of interrelation between WSE (economy in transition) and VSE (developed economy) and their links to FSE. The empirical results, i.e. types and strength of links can serve as indicator of maturity level of WSE

The remainder of the paper is organized as follows. In the second section we give a literature overview about dependence concepts and empirical results. In the next section our main conjectures are formulated. In the fourth section we present the methodology applied. In the fifth section the dataset and empirical results are presented and discussed. The sixth section concludes the paper.

**Literature Overview**

The problem of evaluating the dependence structure between stock markets in a time of globalization is a very important topic. The interrelations between stock markets can be approximated through such variables as stock returns, trading volume and volatility. The simplest methodology in investigations of interdependencies is the causality notion and the Vector Autoregressive (VAR) model. Eun and Shim (1989) investigated relationships between nine large stock markets including those of Australia, Canada, France, Germany, Hong Kong, Japan, Switzerland, the UK and the US by means of the VAR model. They found that the US stock market had the predominant impact on other markets. Lin, Engle, and Ito (1994) checked the interdependence between returns and the volatility of the US and Japanese markets based on high frequency data (daytime and overnight returns). The result was that daytime returns in the US or Japanese market were related to each other’s overnight returns.
Kim and Rogers (1995) studied the dynamic interdependence between the stock markets of the US, Japan and Korea using the multivariate GARCH model. The conclusion was that the Japanese and US stock markets increased their impact on the Korean stock market after its opening to foreign investors. Booth, Martikainen, and Tse (1997) using the E-GARCH model, found strong interdependence among Scandinavian stock markets. Ng (2000) detected one way causality running from the US and Japanese stock markets to six Asian markets, including Hong Kong, Korea, Malaysia, Singapore, Taiwan and Thailand. Lee (comp. Sharkasi, Ruskin, and Crane 2005), by the wavelets technique, found that developed markets (the US, Germany and Japan) had effects on two emerging markets, those of Egypt and Turkey. Antoniou et al. (comp. Sharkasi, Ruskin, and Crane 2005) by the VAR-EGARCH model checked the interdependence between three EU markets: France, Germany and the UK. These results support the notion of cointegration between the stock markets of those countries.

Sharkasi, Ruskin, and Crane (2005) found global co-movements in seven stock markets, three in Europe (namely the Irish, the UK, and Portuguese), two in Americas (the US, and Brazilian) and two in Asia (Japanese and Hong Kong).

Nivet (1997) checked the random walk hypothesis for the Warsaw Stock Exchange. Worthington and Higgs (2004) were concerned with the efficiency of the Hungarian, Polish, Czech and Russian stock markets. The contributors established that only the Hungarian stock market followed the random walk. Gilmore and McManus (2003) found autocorrelations in some stock returns from Central and Eastern European stock markets. In Schotman and Zalewska (2006), the same observation followed from nonsynchronous trading and an asymmetric response to good and bad news.

Todea and Zaicas-Ienciu (2008) investigated the temporal persistence of linear and, especially, nonlinear links between six Central and Eastern European stock markets.

The method, based on extreme value theory, was conducted by Ang and Chen (2002). They drew the conclusion that regime-switching models were most suitable for asymmetry modeling. Regime switching models were introduced into econometrics by Hamilton (1989). Nowadays, they are widely applied in finance. Ang and Bekaert (2002a and 2002b), by the Gaussian Markov switching model, detected two regimes for international returns: a bull regime with a positive mean, low volatilities and
low correlations; and a bear regime with negative returns, high volatilities and correlations.

Patton (2004) detected a significant asymmetry in the dependence of financial returns. Jondeau and Rockinger (2006) applied the skewed-$t$ GARCH model to returns with a univariate time-varying skewness and used a time-varying, a switching Gaussian, or a Student $t$ copula. Hu (2006) suggested replacing the unconditional margins of a copula with conditional margins from univariate GARCH models. This led to a special case of the copula based multivariate dynamic (CMD) model. Klein, Köck, and Tinkl (2010) conducted an extensive simulation study. They suggested that the copula (mis) specification should play a key role before the adaption of a CMD model.


However, there are two serious problems in using copulas. First of all, many of the copulas applied do not have moments that can be directly related to the Pearson correlation. In consequence, it is difficult to compare those results obtained using copulas to those of financial models based on correlations and variances. There is a more essential problem from a statistical point of view. It is not easy to choose a class of parametric copulas which properly fits a given dataset. Some classes of copulas model better near the center and others near the tails of any particular time series distribution. A possible extension to overcome this difficulty is to focus on different shapes of those copulas that are important from a finance perspective, and by using several specification tests which are common in time series analysis. Most contributors do not rigorously justify the choice of particular kinds of copulas.

In their interesting contribution, Qiao, Li, and Wong (2011) combined the multivariate Markov-switching-VAR model developed by Krolzig (1997) and the regime-dependent impulse response analysis technique by Ehrmann, Ellison, and Valla (2003). They investigated dynamic relation-
ships between the stock markets of the US, Australia and New Zealand. The contributors uncovered the existence of two different regimes in the three stock markets. They found that correlations among the three markets were significantly higher in a bear regime than in a bull regime. Moreover, the responses of each of the three markets to shocks in the other two markets were essentially stronger and more persistent in the bear regime than in the bull regime. The authors demonstrated that for the New Zealand stock market, the Australian stock market was more influential than the US stock market, and that for the Australian stock market, the US stock market was more influential than the New Zealand stock market. Our approach, briefly outlined in the introductory section, is somewhat related to that of Qiao, Li, and Wong (2011).

**Main Conjectures**

Taking into account the literature review, the size of the markets under study and economic reasoning, we can formulate some research hypotheses. As we pointed out earlier, the high level of capitalization in the FSE, its maturity and the size of the German economy imply that impulses coming from the FSE play a predominant role in the VSE and the WSE. Therefore we can expect that:

**Conjecture 1** The pairs of indices DAX-ATX and DAX-WIG are likely to be more correlated than the pair ATX-WIG. Moreover, these correlations in a bear phase are stronger than in a bull phase.

The linkage between real economies and stock markets documented in theoretical and empirical studies and the observation that stock markets in a bear phase are more volatile than stock markets in a bull phase have motivated us to formulate the following:

**Conjecture 2** The depth of the recession in an economy will be reflected in the volatility level of its stock market.

In line with the literature, the impulse response function is related to the market phase. Therefore we expect that:

**Conjecture 3** One standard deviation disturbance is higher in a bear market regime than in a bull market. Moreover, the response in the former is more persistent than in the latter.

Taking into account that ATX and WIG are similar in capitalization we predict that:

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Conjecture 4 The responses of the ATX and WIG to one standard deviation disturbance of the DAX are at similar levels.

In the next section of the paper we outline the methodological background.

Methodology

Markov Switching Vector Autoregression

Let \( S_t \) be a two-state unobservable Markov chain. Then, \( X_t \) defined by (1) is driven by a Markov Switching Vector Autoregression model with a lag of length \( p \).

\[
X_t = \begin{cases} 
  v_1 + \sum_{i=1}^{p} A^1_i X_{t-i} + Q_1 u_t & \text{if } S_t = 1 \\
  v_2 + \sum_{i=1}^{p} A^2_i X_{t-i} + Q_2 u_t & \text{if } S_t = 2 
\end{cases}
\] (1)

where, for \( j = 1, 2 \), the intercept term is denoted by \( v_j \), for \( i = 1, \ldots, p \), autoregression terms are denoted by \( A^j_i \), disturbances are represented by \( Q_j u_t \), where \( u_t \sim N(0, I) \) and \( Q_j \) is a matrix generating the covariance matrix. Generalization of the model (1) to more than two regimes is possible, although in this article only two regimes are considered. Clearly, in this model the intercept term and the autoregression terms are regime dependent. Moreover, the variance-covariance matrix \( \Sigma_j \) in the regime \( j = 1, 2 \) takes the following form:

\[
\sum_j = E(Q_j u_t u'_t Q'_j) = Q_j Q'_j. 
\] (2)

Therefore the variance-covariance matrix is regime dependent. The hidden process \( S_t \) is specified by transition probabilities \( p_{ij} \), where, for \( i, j = 1, 2 \),

\[
p_{ij} = P(S_{t+1} = j | S_t = i). 
\] (3)

All transition probabilities form a transition matrix \( P \) defined by:

\[
P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},
\] (4)

where \( p_{11} + p_{12} = p_{21} + p_{22} = 1 \).

The estimation procedure for the model is conducted by the Expectation-Maximization (EM) algorithm. Firstly, the optimal inference \( \xi_t := P(S_t = j) \) is estimated for chosen starting parameters in the expectation
step. Note that $\xi_t$ is a two-dimensional vector. Formally, $\xi_{j,t}$ are being estimated using the Hamilton filter (see Hamilton 1990):

$$\hat{\xi}_{t|t} = \hat{\xi}_{t|t-1} \odot \eta_t,$$  \hspace{1cm} (5)

$$\hat{\xi}_{t+1|t} = P^T \hat{\xi}_{t|t},$$  \hspace{1cm} (6)

where $\eta_t$ denotes the vector of the conditional density under both regimes, $\hat{\xi}_{t|t} = (P[S_t = j|X_t])_{j=1,2}$ and $\hat{\xi}_{t+1|t} = (P[S_{t+1} = j|X_t])_{j=1,2}$ the Hadamard’s multiplication denoted by $\odot$ means the multiplication coordinate by coordinate.

Secondly, the optimal set of parameters, for the estimated $\xi_t$, is found in the maximization step. The Maximum Likelihood (ML) estimation in the case of Markov Switching VAR is equivalent to the Least Square estimation weighted by the estimated $\xi_t$. These steps are repeated until the parameters converge.

Note that, the estimation procedure, for $j = 1, 2$ provides us with estimates of variance-covariance matrices $\Sigma_j$, and matrices $Q_j$ are chosen as lower triangular matrices fulfilling equation (2). Technically, we use the Cholesky-Banachiewicz algorithm to obtain the estimate of $Q_j$ for an estimate of $\Sigma_j$, for $j = 1, 2$.

**UNCONDITIONAL PROPERTIES OF A VARIABLE DRIVEN BY THE REGIME SWITCHING VAR MODEL**

The descriptive statistics of the data illustrate the properties of those data in the most basic way. Moreover, the interpretation of the descriptive statistics is clear, so that using it for a preliminary analysis is convenient. The unconditional characteristics of a series driven by a particular model determine the descriptive statistics of its realization. It is desirable to have the possibility of modeling certain characteristics by the chosen model. For instance, financial time series are usually characterized by a negative skewness and relatively high kurtosis. The chosen model therefore should incorporate these characteristics into its unconditional distribution.

Clearly, the unconditional distribution of the process $X_t$ defined in (1) is a mixed normal distribution. In particular, for some $Z_1 \sim N(\mu_1, \Sigma_1)$ and $Z_2 \sim N(\mu_2, \Sigma_2)$, unconditionally:

$$X_t = (2 - S)Z_1 + (S - 1)Z_2,$$  \hspace{1cm} (7)

where $S$ is a Bernoulli distributed variable representing the unconditional
ms state process, assuming that the ms process is ergodic, $P(S = 1) = \omega_1 = p_{21}/(p_{12} + p_{21})$ and $P(S = 2) = \omega_2 = p_{12}/(p_{12} + p_{21})$; for $i = 1, 2$; the conditional mean and the covariance matrix on being in state $i$ is denoted by $\mu_i$ and $\Sigma_i$, respectively. Note that the mixture of normal distributions is not the same as a linear combination of normal distributions. In particular, the mixture of normal distributions is not normally distributed. We prove this statement in Appendix. It follows from the results in the Appendix that the distribution of the variable driven by model (1) is a mixture of normal multivariate distributions. The relation between the parameter $\mu_1, \mu_2, \Sigma_1, \Sigma_2$ set and the parameters of model (1) is presented in the Appendix. The margins of a multivariate mixed normal distribution have corresponding univariate mixed normal distributions. To be precise, the $d$-th coordinate of $X_t$ is a variable whose distribution is a mixture of normal distributions $N(\mu_{d,1}, \sigma_{d,1}^2)$ and $N(\mu_{d,2}, \sigma_{d,2}^2)$, where the mean $\mu_{d,i}$ is the $d$-th coordinate of $\mu_i$ and $\sigma_{d,i}^2$ is the corresponding Schur complement of the matrix $\Sigma_i$ for $i = 1, 2$.

Combining the calculation results presented in the Appendix, for a given parameter set of model (1), we are able to calculate the theoretical characteristics of the realization. It is possible to obtain multivariate characteristics such as mean (see formula (20)) and covariance matrix (see formula (30)) and interesting univariate ones, like skewness (see formula (16)) and kurtosis (see formula (17)).

Clearly, it is more convenient to simulate a realization of the process and calculate characteristics based on this realization which will be precise enough if the simulation is long enough. However, the presented method is analytical and additionally illustrates how parameters affect skewness and kurtosis. For instance, the skewness (see formula (16)) is significantly negative for $\sigma_{1}^2 < \sigma_{2}^2, \mu_1 > \mu_2$ and $\omega_2 < \omega_1$. Let the first state correspond to a time of prosperity and the second to one of recession. Intuition, along with theoretical and empirical evidence confirm that the variance is higher in the latter, while the mean is higher in the former, and also that periods of prosperity last longer than those of crisis.

**IMPULSE RESPONSE FUNCTION IN THE REGIME SWITCHING MODEL**

The impulse response function (irf) is a very convenient tool in measuring dependencies between financial variables. Intuitively, the irf presents the influence of a disturbance in one variable on others. The
response $\Psi^k_i$ to a disturbance in the $k$-th variable conditional on being in regime $j$ during a whole period is defined by:

$$
\Psi^k_j(h) := \frac{\partial E_t X_{t+h}}{\partial u_t^k} \bigg|_{s_t=s_{t+1}=\cdots=s_{t+h}=j}, \quad h > 0 \quad (8)
$$

where $u^t$ is the vector of zeros apart from the $k$-th element which is one. In the case of VAR, the IRF defined in (8) is estimated as follows:

$$
\Psi^k_j(0) = \hat{Q}_j u_0, \quad (9)
$$

$$
\Psi^k_j(h) = \max(h,p) \sum_{i=1}^{h-i+1} A_i^j \hat{Q}_j u_0 \quad (10)
$$

It is inadequate to use formula (8) in the case of a long time horizon or an insufficiently persistent regime process $s_t$. In this case, the path $s_t = s_{t+1} = \cdots = s_{t+h} = j$ is improbable. The IRF conditional on starting in a particular regime may be more informative value. The response to the disturbance in the $k$-th variable conditionally on starting in regime $j$, denoted by $\Xi^k_j$, is defined by:

$$
\Xi^k_j(h) := \frac{\partial E_t X_{t+h}}{\partial u_t^k} \bigg|_{s_t=j}, \quad h > 0 \quad (11)
$$

As in formulas (9) and (10), in the case of MSVAR, the IRF defined in (11) is estimated as follows:

$$
\tilde{\Xi}^k_j(0) = \hat{Q}_j u_0, \quad (12)
$$

$$
\tilde{\Xi}^k_j(h) = \max(h,p) \sum_{i=1}^{h-i+1} \sum_{J=\{j_1=j, j_2, \ldots, j_h\} \in \{1,2\}^h} p_{j_i=j} \left( \prod_{v=1}^{h-i+1} \hat{A}_{j_v} \right) \hat{Q}_j u_0, \quad (13)
$$

where $p_{j_i=j}$ denotes the probability of path $J$ conditionally on starting at $j$. Note that in formulas (10) and (13), for $i < 1$ and $i > p$, matrix $A_i^j$ equals 0.

The evaluation of error for an IRF, in the case of regime switching models, is carried out using the bootstrap method. In order to estimate the distribution of an IRF, the following five step procedure is to be repeated a sufficient number of times:

1. According to formulas (3) and (4), simulate the history of state process $S_t$, recursively. The elements of the transition matrix are replaced by the corresponding estimates.
Table 1: Descriptive Statistics for Percentage Logarithmic Weekly Returns

<table>
<thead>
<tr>
<th>Item</th>
<th>DAX</th>
<th>ATX</th>
<th>WIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.2063</td>
<td>0.1380</td>
<td>0.1317</td>
</tr>
<tr>
<td>Median</td>
<td>0.4077</td>
<td>0.5449</td>
<td>0.1726</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>3.2989</td>
<td>3.6951</td>
<td>3.4388</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.5303</td>
<td>-1.8996</td>
<td>-0.2956</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.3951</td>
<td>18.2820</td>
<td>7.3598</td>
</tr>
<tr>
<td>Minimum</td>
<td>-18.8700</td>
<td>-34.1301</td>
<td>-19.9103</td>
</tr>
<tr>
<td>Maximum</td>
<td>19.8639</td>
<td>17.2196</td>
<td>19.7664</td>
</tr>
</tbody>
</table>

2. Following formula (1), simulate the history of endogenous variable $X_t$, recursively. Matrices occurring in (1) are replaced by the corresponding estimates.

3. For the generated process, conduct the estimation procedure presented below. The procedure yields new estimates of autoregression matrices $\{\hat{A}_j\}_{j=1,2}$, the covariance matrix for errors $\{\hat{\Sigma}_j\}_{j=1,2}$, the transition matrix $\hat{P}$ and the smoothed probabilities $\{\hat{\xi}_t\}_{t=1,\ldots,T}$.

4. Using the same procedure as for the primary estimation, for $j = 1, 2$, calculate matrix $\hat{Q}_j$ for matrix $\hat{\Sigma}_j$ obtained in step 3.

5. Using formulas (9) and (10) or (12) and (13), calculate estimates of the IRF for the bootstrapped parameters obtained in steps 3 and 4.

The approximation of the distribution of the IRF consists of the $N$ sets of estimates obtained in step 5, where $N$ is the number of repetitions of the procedure.

**Dataset and Empirical Results**

**Description of the Data**

The dataset consists of the prices of three stock market indices that is to say the German DAX30, the Austrian ATX20 and the Polish WIG20. Wednesday to Wednesday weekly returns are used in the analysis. Compared to daily returns, weekly return processes have lower autocorrelation and avoid the missing data problem. Moreover, VAR based models work better with smoother weekly data than with noisier daily data. This gives us a sample of 571 weekly returns from January 2003 to December 2013. We apply continuous logarithmic percentage returns.

Firstly, we present some descriptive statistics in table 1.
In the period under study we observe positive means in all three indices. The relatively high absolute value of median and negative skewness suggest asymmetries in these time series. As we stated in the previous section and calculated in the Appendix, model (1) can generate a process with negative skewness and large kurtosis.

**MARKOV SWITCHING VAR ESTIMATION RESULTS**

Following the notation presented in (1), the estimation results of autoregressive matrices and covariance matrices for the data are presented in table 2. The estimation procedure provides us with two regimes of the VAR model of order two, this order being chosen due to information criteria. In fact, the information criteria for three lags are similar to those of two lags, although it is more convenient to present and interpret VAR estimates of order two. We have conducted a complete analysis of the MSVAR of order three and the conclusions appear to be similar.

The first regime, estimates of which are presented on the left side of table 2, is characterized by negative means for every index, and high volatility. Conversely, the second regime (right side of table 2) is characterized by positive means and relatively small volatility. Additionally, in table 3, we present correlations of the series in both regimes. The correlations are higher in the first regime for all three pairs. These properties clearly describe a bull market (the second regime) and a bear market (the first regime).

There is little point, in terms of information, in analyzing all differences between the parameters sets in the two regimes. We observe similar signs in the majority of autoregression parameters (coefficients of the $A$ matrices) for both regimes. The analysis of the impulse response function presented in the next section is far more practical and informative. An interesting finding is that the highest volatility in the second regime is found for the WIG return process, while it is the least volatile variable in the first regime. A possible explanation of this unexpected finding is that during this whole period, the rate of growth in Poland was positive, while in Germany and Austria there was a recession (especially in 2009).

The stationarity of residual series is essential in IRF analysis. We have performed Augmented Dickey-Fuller (ADF) test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for each residual of the estimated model. Both tests confirm stationarity of residuals, the null hypothesis of ADF test is rejected on any reasonable significance level for each of the

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### TABLE 2  Estimation Results of the Regime

<table>
<thead>
<tr>
<th>Item</th>
<th>Bear market regime</th>
<th>Bull market regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DAX</td>
<td>ATX</td>
</tr>
<tr>
<td>(v_j)</td>
<td>-0.0006</td>
<td>-0.0022</td>
</tr>
<tr>
<td>(A_1^j)</td>
<td>-0.2723</td>
<td>0.1295</td>
</tr>
<tr>
<td></td>
<td>0.0274</td>
<td>-0.1736</td>
</tr>
<tr>
<td></td>
<td>0.1748</td>
<td>0.1181</td>
</tr>
<tr>
<td>(A_2^j)</td>
<td>0.1074</td>
<td>0.0094</td>
</tr>
<tr>
<td></td>
<td>0.1082</td>
<td>0.1153</td>
</tr>
<tr>
<td></td>
<td>-0.2351</td>
<td>-0.1816</td>
</tr>
<tr>
<td>(\sigma_j^2)</td>
<td>0.0021</td>
<td>0.0027</td>
</tr>
</tbody>
</table>

**Notes**: \(v_j\) is the intercept term, \(A_1^j\) and \(A_2^j\) denote the first and second lag autoregression matrices, respectively; and \(\sigma_j^2\) is the diagonal of the variance-covariance matrix, where \(j\) denotes the regime.

### TABLE 3  Regime Dependent Correlations

<table>
<thead>
<tr>
<th>Item</th>
<th>DAX-ATX</th>
<th>DAX-WIG</th>
<th>ATX-WIG</th>
</tr>
</thead>
<tbody>
<tr>
<td>First regime</td>
<td>0.7341</td>
<td>0.6938</td>
<td>0.6006</td>
</tr>
<tr>
<td>Second regime</td>
<td>0.6356</td>
<td>0.5292</td>
<td>0.5168</td>
</tr>
</tbody>
</table>

**Note**: Correlations of the variance-covariance matrix in both regimes.

Three variables. The null hypothesis for the KPSS test is not rejected on 5%-significance level for any residual.

**Impulse Response Function Analysis**

Responses to DAX impulses are the most important from a practical point of view. The German stock market is much bigger and much more important than the two other markets. The impulse response functions in both regimes are essentially different. The one standard deviation disturbance is much higher in the first regime. The values of the impulse response functions are even more significant. Moreover, the period in which the response is significant is longer in the first regime. The response of the ATX in the first regime is positive in the first week after a DAX disturbance and even stronger in the second week. In the second regime, the second week response of the ATX to DAX disturbance is lower than the first week and close to zero. In the case of the WIG, we observe an interesting relationship in the first regime. After the positive first week response, there is a negative one, with relatively high abso-


### Table 4: Confidence Interval of the Impulse Response Function in the First Regime

<table>
<thead>
<tr>
<th>Item</th>
<th>Endpoint</th>
<th>Item</th>
<th>Endpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>Upper</td>
<td>0.0573</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.0351</td>
<td>1</td>
</tr>
<tr>
<td>ATX</td>
<td>Upper</td>
<td>0.0000</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.0000</td>
<td>1</td>
</tr>
<tr>
<td>WIG</td>
<td>Upper</td>
<td>0.0000</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.0000</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note**: Upper and lower endpoint of the 90% confidence interval of the impulse response function in the bear market regime.

### Table 5: Confidence Interval of the Impulse Response Function in the Second Regime

<table>
<thead>
<tr>
<th>Item</th>
<th>Endpoint</th>
<th>Item</th>
<th>Endpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>Upper</td>
<td>0.0223</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.0156</td>
<td>1</td>
</tr>
<tr>
<td>ATX</td>
<td>Upper</td>
<td>0.0000</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.0000</td>
<td>1</td>
</tr>
<tr>
<td>WIG</td>
<td>Upper</td>
<td>0.0000</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>lower</td>
<td>0.0000</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note**: Upper and lower endpoint of the 90% confidence interval of the impulse response function in the bull market regime.

The impulse response analysis can provide investors with very valuable short-run prognoses. In a time of prosperity (the second regime), the impulse response function process is in line with economic expectations. Apart from the significant difference in values for both regimes, we observe an unexpected IRF path. In particular, it is possible that a pos-
itive disturbance causes another variable to have negative values in the future.

The estimated switching process is not very persistent, following the notation presented in (3), $p_{11} = 0.82$ and $p_{22} = 0.89$. However, the expected time for shifting is nearly 6 in the first regime and greater than nine in the second regime. The expected time for shifting $d_j$ in the regime $j$, for $j = 1, 2$, equals $1/(1 - p_{jj})$. In table 5, we see that the IRF for lags greater than 3 diverges insignificantly from zero, so that the IRF defined in (11) is similar to the IRF defined in (8), so we have not presented these results.

**Concluding Remarks**

Empirical results for German, Austrian and Polish markets provide evidence that in a bear market regime correlations among pairs of indices are essentially stronger that in a bull market regime. In addition, these correlations depend on the size of markets. They are more pronounced in the case of the pairs ATX-DAX and WIG-DAX than the ATX-WIG. These findings are in line with the first conjecture.

The financial literature indicates that changes in volatility depend on events which are important to a particular stock market. In addition, negative information in announcements is the source of higher volatility than release of positive information. Moreover, it is well known that emerging stock markets are characterized in general by high volatility, so it is not surprising that the highest volatility in the second regime was estimated for WIG returns. However, WIG is the least volatile index in the first regime. At first sight, this finding contradicts the common conviction, widely represented in the financial literature, that the more mature the market, the lower the volatility. In order to explain this observation, we have to take into account that during this whole analyzed period, the rate of growth of the Polish economy was positive, while in Germany and Austria there was a recession (especially in 2009). This fact is a possible reason for the surprising ranking of the volatilities of markets under study in the bear phase. These findings support the second research hypothesis.

Impulse response functions in both regimes are essentially different. The first observation is that one standard deviation disturbance is much higher in the first regime. In addition, the persistence of the impulse response is more pronounced in this regime. This observation supports the third conjecture. The response of the ATX in the first regime is positive in the first week after a DAX disturbance and it is even stronger in the second week. By contrast, in the second regime, the second week re-
response of the \textit{ATX} to the \textit{DAX} is lower than that of first week, and very close to zero. In the case of the \textit{WIG}, see an interesting relationship in the first regime. After a positive first week response, there is a negative and relatively high in absolute value one in the second week. In the second regime, the response of the \textit{WIG} is very similar to the response of the \textit{ATX} to \textit{DAX} disturbances. These findings are only partly in line with the fourth conjecture.

To summarize, these results indicate the predominant role of the \textit{DAX} index among the three markets. Most results are in line with expectations. However, some of them, for instance, the ranking of volatilities in the first (bear market regime), are surprising. The impulse response function is shown to be an important complementary tool in the testing of market reactions to shocks on both domestic and foreign markets, which is done in the short run context in this contribution. This tool may be essential with respect to the prediction of the very important contagion effect on financial markets especially in a bear phase of world stock markets. Another important issue is the assessment of persistency and under- and overreaction related to the market phase.

The last but not least important result (see the Appendix) is the theoretical unconditional distribution of a process driven by the Markov switching \textit{VAR} model. We calculated unconditional mean, variance, skewness and kurtosis for the margins of the process. The results illustrate how the parameters of the \textit{MSVAR} model affect unconditional characteristics and prove that within this model it is possible to model asymmetric variables with unlimited kurtosis. The results can be used to calculate theoretical unconditional characteristics. Having said that, we find it rather inconvenient and prefer to perform calculations using the Monte Carlo Method.

\textbf{Appendix}

Let us compute the mean, variance, and third and the fourth central moments of variable with a mixed univariate normal distribution. Let \( X = (2 - S)Z_1 + (S - 1)Z_2 \), where \( Z_1 \sim N(\mu_1, \sigma_1) \) and \( Z_2 \sim N(\mu_2, \sigma_2) \). We use a notation: \( P(S_t = 1) = \omega_1 \) and \( P(S_t = 2) = \omega_2 \), such that \( \omega_1 + \omega_2 = 1 \). In the case of model (1), following notation (3), \( \omega_1 = p_{21}/(p_{12} + p_{21}) \) and \( \omega_2 = p_{12}/(p_{12} + p_{21}) \).

\[
E(X) = E(E(X|S)) = \omega_1 E(X|S = 1) + \omega_2 E(X|S = 2) = \omega_1 \mu_1 + \omega_2 \mu_2.
\]

Let us denote the mean by \( \mu \), thus \( \mu = \omega_1 \mu_1 + \omega_2 \mu_2 \) and for \( i = 1, 2 \), the conditional variable \( X_i = (X|S = i) \). The \( j \)-th central moment of variable

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X is computed as follows:

\[
E((X - \mu)^j) = E(E((X - \mu)^j | S)) = \omega_1 E((X - \mu)^j | S = 1) + \omega_2 E((X - \mu)^j | S = 2)
\]

\[
= \omega_1 E((X - \mu_1 + \mu_1 - \mu)^j | S = 1) + \omega_2 E((X - \mu_2 + \mu_2 - \mu)^j | S = 2)
\]

\[
= \omega_1 \sum_{k=0}^{j} \binom{j}{k} (\mu_1 - \mu)^j E((X_1 - \mu_1)^k)
\]

\[
+ \omega_2 \sum_{k=0}^{j} \binom{j}{k} (\mu_2 - \mu)^j E((X_2 - \mu_2)^k).
\]

Therefore, the variance takes the following form:

\[
D^2(X) = E((X - \mu)^2) = \sum_{i=1}^{2} \omega_i \sum_{k=0}^{2} \binom{2}{k} (\mu_i - \mu)^{2-k} E((X_i - \mu_i)^k)
\]

\[
= \sum_{i=1}^{2} \omega_i ((\mu_i - \mu)^2 + \sigma_i^2)
\]

\[
= \omega_1 (\omega_2^2 (\mu_1 - \mu_2)^2 + \sigma_1^2) + \omega_2 (\omega_2^2 (\mu_2 - \mu_1)^2 + \sigma_2^2)
\]

\[
= \omega_1 \sigma_1^2 + \omega_2 \sigma_2^2 + \omega_1 \omega_2 (\mu_1 - \mu_2)^2.
\]

The third central moment is computed as follows:

\[
E((X - \mu)^3) = \sum_{i=1}^{2} \omega_i \sum_{k=0}^{3} \binom{3}{k} (m\mu_i - \mu)^{3-k} E((X_i - \mu_i)^k)
\]

\[
= \sum_{i=1}^{2} \omega_i ((\mu_i - \mu)^3 + 3(\mu_i - \mu) E((X_i - \mu_i)^2) + \sigma_i^3)
\]

\[
= \omega_1 \omega_2 (\omega_1 + \omega_2) (\omega_2 - \omega_1) (\mu_1 - \mu_2)^3
\]

\[
+ 3 (\mu_1 - \mu_2) (\sigma_1^2 - \sigma_2^2)
\]

\[
= \omega_1 \omega_2 (\mu_1 - \mu_2) (\omega_2 - \omega_1) (\mu_1 - \mu_2)^2
\]

\[
+ 3 (\sigma_1^2 - \sigma_2^2).
\]

Finally, let us compute the fourth central moment:

\[
E((X - \mu)^4) = \sum_{i=1}^{2} \omega_i \sum_{k=0}^{4} \binom{4}{k} (\mu_i - \mu)^{4-k} E((X_i - \mu_i)^k)
\]
\[
E(X_t) = v_1 + \sum_{j=1}^P \sum_{i \in \{1,2\}^j} p_{j(i)} A_j^{I(j)} E(X_{I(j)}) \\
E(X_2) = v_2 + \sum_{j=1}^P \sum_{i \in \{1,2\}^j} p_{j(i)} A_j^{I(j)} E(X_{I(j)})
\]

\[
\begin{align*}
\mu &= E(X) = \omega_1 \mu_1 + \omega_2 \mu_2; \\
\sigma^2 &= D^2(X) = \omega_1 \sigma_1^2 + \omega_2 \sigma_2^2 + \omega_1 \omega_2 (\mu_1 - \mu_2)^2; \\
\gamma_1 &= \frac{E(X - \mu)^3}{\sigma^3} = \\
&= \frac{\omega_1 \omega_2 (\mu_1 - \mu_2)((\omega_2 - \omega_1)(\mu_1 - \mu_2)^2 + 3(\sigma_1^2 - \sigma_2^2))}{\left(\sqrt{\omega_1 \sigma_1^2 + \omega_2 \sigma_2^2 + \omega_1 \omega_2 (\mu_1 - \mu_2)^2}\right)^3}; \\
\beta_2 &= \frac{E(X - \mu)^4}{\sigma^4} = \\
&= \frac{3\omega_1 \sigma_1^4 + 3\omega_2 \sigma_2^4}{(\omega_1 \sigma_1^2 + \omega_2 \sigma_2^2 + \omega_1 \omega_2 (\mu_1 - \mu_2)^2)^2} \\
&+ \frac{\omega_1 \omega_2 (\mu_1 - \mu_2)^4((\omega_2^3 + \omega_1 \omega_2 + \omega_2^2)(\mu_1 - \mu_2)^4)}{(\omega_1 \sigma_1^2 + \omega_2 \sigma_2^2 + \omega_1 \omega_2 (\mu_1 - \mu_2)^2)^2} \\
&+ \frac{6(\omega_2 \sigma_2^2 + \omega_1 \sigma_2^2)}{(\omega_1 \sigma_1^2 + \omega_2 \sigma_2^2 + \omega_1 \omega_2 (\mu_1 - \mu_2)^2)^2};
\end{align*}
\]

where \(\sigma^2\) denotes the unconditional variance of the variable, \(\gamma_1\) denotes the unconditional skewness of \(X\) and kurtosis is denoted by \(\beta_2\).

The parameter set \(\mu_1, \mu_2, \sigma_1, \sigma_2\) of the variable \(X_t\) presented in (7) can be written in terms of the parameters of model (1). Let us introduce an additional notation, for \(j \in \mathbb{N}\) and \(I \in \{1,2\}^j\), we denote \(p_I := \pi_i^j = p_{I(i-1)I(i)}\), where \(I(i)\) denotes the \(i\)-th coordinate of the \(I\). Intuitively, \(p_I\) is the probability of the path \(I\) for a Markov chain \(S_t\) with a transition matrix (4). For now on, we denote \(X_t := (X_t|S_t = i)\). Taking the expected value in the equation (1), we obtain the following system of linear equations:

\[
\begin{align*}
E(X_1) &= v_1 + \sum_{j=1}^P \sum_{I \in \{1,2\}^j} p_{(i)} A_j^{I(j)} E(X_{I(j)}) \\
E(X_2) &= v_2 + \sum_{j=1}^P \sum_{I \in \{1,2\}^j} p_{(i)} A_j^{I(j)} E(X_{I(j)})
\end{align*}
\]

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where \((I, i)\), for \(i \in \{1, 2\}\) and \(I \in \{1, 2\}^{j+1}\), denotes \(J \in \{1, 2\}^{j+1}\) such that \(J(k) = I(k)\), for \(k = 1, \ldots, j\), and \(J(j+1) = i\). Transforming (18), we have:

\[
\begin{cases}
(1 - \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j} P_{(1,I,i)} A_j^1) E(X_1) \\
= v_1 + \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j-1} P_{(2,I,i)} A_j^2 E(X_2) \\
(1 - \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j} P_{(2,I,i)} A_j^1) E(X_2) \\
= v_2 + \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j-1} P_{(1,I,i)} A_j^1 E(X_1)
\end{cases}
\]

therefore

\[
\begin{cases}
E(X_1) = (\Xi_1 + \Psi_1 \Xi_2^{-1} \Psi_2)^{-1}(v_2 + \Psi_1 \Xi_2^{-1} v_2) \\
E(X_2) = (\Xi_2 + \Psi_2 \Xi_1^{-1} \Psi_1)^{-1}(v_1 + \Psi_2 \Xi_1^{-1} v_1)
\end{cases} \quad (19)
\]

hence,

\[
E(X_t) = \omega_1 E(X_1) + \omega_2 E(X_2) \\
= \omega_1 (\Xi_1 + \Psi_1 \Xi_2^{-1} \Psi_2)^{-1}(v_2 + \Psi_1 \Xi_2^{-1} v_2) \\
+ \omega_2 (\Xi_2 + \Psi_2 \Xi_1^{-1} \Psi_1)^{-1}(v_1 + \Psi_2 \Xi_1^{-1} v_1), \quad (20)
\]

where

\[
\Xi_1 : = 1 - \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j} P_{(1,I,i)} A_j^1, \quad (21)
\]

\[
\Xi_2 : = 1 - \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j} P_{(2,I,i)} A_j^2, \quad (22)
\]

\[
\Psi_1 : = \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j-1} P_{(2,I,i)} A_j^2, \quad (23)
\]

\[
\Psi_2 : = \sum_{j=1}^{p} \sum_{I \in \{1, 2\}}^{j-1} P_{(1,I,i)} A_j^1. \quad (24)
\]

System (19) shows us how to compute \(\mu_1: = E(X_1)\) and \(\mu_2: = E(X_2)\).

In order to compute \(\sum_1\) and \(\sum_2\), an additional notation is needed. Let, for \(i = 1, 2,\)

\[
X_{i,t}^* : = ([X_{t}', \ldots, X_{t-p}'][S_t = i]). \quad (25)
\]
Then, rewriting (1), we have:

\[ X_{i,t}^* = v_i^* + \sum_{j=1}^{2} \Xi_{i,j}^* X_{j,t-1}^* + \epsilon_{i,t}^*, \ i = 1, 2, \] (26)

where

\[ v_i^* := \begin{bmatrix} v_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ i = 1, 2; \] (27)

\[ \Xi_{i,j}^* := \sum_{I \in \{1,2\}^p} p(I,i) \begin{bmatrix} \chi_j(I(p))A_i^j \\ \chi_j(I(p-1))A_2^j \\ \vdots \\ \chi_j(I(1))A_p^j \end{bmatrix}, \ i = 1, 2; \] (28)

\[ \epsilon_{i,t}^* := \begin{bmatrix} Q_{i,tt} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \ i = 1, 2; \] (29)

Therefore, for \( i = 1, 2, \)

\[ X_{i,t}^* = v_i^* + \sum_{j=1}^{2} \Xi_{i,j}^* X_{j,t-1}^* + \epsilon_{i,t}^* \]

\[ = v_i^* + \sum_{j=1}^{2} \Xi_{i,j}^* (v_{j,t}^* + \sum_{j_2=1}^{2} \Xi_{j_2}^* X_{j_2,t-2}^* + \epsilon_{j_2,t-1}^*) + \epsilon_{i,t}^* \]

\[ = v_i^* + \epsilon_{i,t}^* + \sum_{j_1=1}^{2} \Xi_{i,j_1}^* (v_{j_1,t}^* + \epsilon_{j_1,t-1}^*) \]

\[ + \sum_{j_1=1}^{2} \sum_{j_2=1}^{2} \Xi_{i,j_1}^* \Xi_{j_2}^* (v_{j_2,t}^* + \sum_{j_3=1}^{2} \Xi_{j_3}^* X_{j_3,t-3}^* + \epsilon_{j_3,t-2}^*) \]
\[
= \cdots = v_i^* + \varepsilon_{i,t}^* + \sum_{j=1}^{\infty} \sum_{l(i)=i} \left( \prod_{k=1}^{j} \Xi_{l(k),I(k+1)}^* \right) (v_{l(j+1)}^* + \varepsilon_{l(j+1),t-j}^*).
\]

The \(\varepsilon_{i,t-j}^*\) are iid, thus we have:

\[
\sum_i: = \text{Var}(X_{i,t}^*) = Q_i Q_i' + \sum_{j=1}^{\infty} \Psi_{i,j} Q_i Q_i' \Psi_{i,j}', i = 1, 2, \quad (30)
\]

where

\[
\Psi_{i,j}^* = \sum_{l(i) + 1}^{l(i)} \left( \prod_{k=1}^{j} \Xi_{l(k),I(k+1)}^* \right), i = 1, 2, j \in \mathbb{N}. \quad (31)
\]

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