

# *Privatization and Optimum-Welfare in an International Cournot Duopoly*

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In this paper, we will analyse the relationship between privatization of a public firm and tax revenue for the domestic government in an international competition, with import tariffs. We consider a duopoly model where a domestic public firm and a foreign private firm compete in the domestic market, as Cournot players. Furthermore, the domestic government imposes a tariff to regulate an imported good, and may have a higher preference for tariff revenue than for social welfare. We compute the outputs at equilibrium and we show that privatization (i) will increase the profits of both domestic and foreign firms; (ii) will increase the tariff imposed to the imported good; and (iii) will decrease the domestic welfare. Furthermore, we demonstrate that a rise in the government's preference for tariff revenues raises the social welfare in both mixed and private models.

*Key Words:* game theory, industrial organization, Cournot model, privatization

*JEL Classification:* C72, L33

## **Introduction**

Mixed oligopolies, where a public firm competes with private firms are common in developed and developing countries. Public firms compete against private firms in many industries, such as airlines, banking, electricity, steel, and so on.

Ferreira and Ferreira (2014) analysed the relationship between the privatization of a public firm and government preferences for tax revenue in a duopoly model, by assuming that the government payoff is given by a weighted sum of tax revenue and the sum of consumer and producer surplus. De Fraja and Delbono (1989) showed that privatization of the public firm may improve social welfare. Pi, Yang, and Zhou (2013) and

Ferreira and Ferreira (2016) investigated the economic impacts exerted by privatization in a model that considers environmental problems. They showed that, when the residents' environmental preference is introduced to the public firm's payoff function, privatization may increase the public firm's output, decrease the private firm's output and has no effect on social welfare. White (1996) studied effects of domestic production subsidies in a mixed oligopoly market. Pal and White (1996) investigated the effects of privatization in the presence of strategic trade policies in a market with foreign firms. In a mixed duopoly, Matsumura and Ogawa (2010) analysed whether private leadership or public leadership is robust in the observable delay game. They showed that private leadership is more robust than public leadership. Some authors examine price-setting mixed market models (see, for instance, Bárcena-Ruiz 2007 and Ohnishi 2011).

Many works on mixed oligopoly analysed domestic competitions. However, more recent literature in this field of industrial organization includes foreign private firms. The introduction of foreign firms affects, or may affect the results, since the social welfare does not include the producer surplus of the foreign firms (see Fjell and Pal 1996). Lee, Xu, and Chen (2013) showed that the equilibrium degree of privatization depends not only on the relative efficiency of the public firm, but also on trade policy. Moreover, they also demonstrated that competitive privatization with a tariff achieves a higher degree of privatization than without a tariff.

In this paper, we analyse the effects of privatization in a domestic market served by a domestic public firm and a foreign private firm, by assuming that the government imposes an import tariff. Furthermore, we allow the government may prefer tariff revenue more than social welfare. We show that increasing the government's preference for tariff revenues leads to reduction on the aggregate quantity in the mixed market, while the opposite is true after privatization of the public firm. Furthermore, a rise in government's preference for tariff revenues raises domestic public firm's profit; but it lowers domestic privatized firm's profit. Moreover, as the government's preference for tariff revenues increases, the social welfare becomes higher in both mixed and private models.

The remained of the paper is organised as follows. Section 2 describes the mixed model in which the government may prefer tariff revenue to the sum of consumer and producer surplus. In Section 3, we compute and analyse the equilibrium outcome. Section 4 solves the privatized model. In Section 5, we do a comparative static analysis. Section 6 compares the two models. Section 7 concludes the paper.

### The Mixed Model

We consider an international mixed duopoly with one domestic public firm  $F_d$  and one foreign private firm  $F_f$ . We assume that both firms produce a homogeneous good and the market demand is given by

$$p = 1 - q_d - q_f,$$

where  $p$  is the price,  $q_d$  is the quantity produced by the domestic public firm and  $q_f$  is the quantity produced by the foreign private firm. Both firms have the same quadratic cost function

$$C(q_i) = \frac{q_i^2}{2}, \quad i = d, f.$$

Furthermore, we assume that the government of the domestic country imposes a tariff  $t$  on imported goods.

So, each firm's profit function is defined by

$$\begin{aligned} \pi_d &= (1 - q_d - q_f)q_d - \frac{q_d^2}{2}, \\ \pi_f &= (1 - q_d - q_f)q_d - \frac{q_f^2}{2} - tq_f. \end{aligned}$$

The domestic public firm chooses its output that maximizes the sum of consumer's surplus and domestic producer surplus:

$$W = CS + PS = \frac{1}{2}(q_d = q_f)^2 + \pi_d.$$

The government's payoff is given by

$$U = W + (1 + \alpha)R,$$

where  $R = tq_f$  and  $\alpha$  is the parameter that represents the weight of the government's preference for tariff revenues. We consider  $\alpha \geq 0$ , i.e., the government values tariff revenues  $R$  more than social welfare  $W$ .

The timing of the game is as follows:

- In the first stage, the government sets the import tariff  $t$ ;
- In the second stage, each firm, independent and simultaneously, chooses its quantity  $q_i$ ,  $i = d, f$ , knowing already the import tariff imposed by the government.

### Analysis of the Mixed Model

To obtain a subgame perfect equilibrium, the game is solved by backwards induction.

So, we differentiate the function  $W$  with respect to  $q_d$  and the function  $\pi_f$  with respect to  $q_f$ :

$$\begin{aligned}\frac{\partial W}{\partial q_d} &= 1 - 2q_d = 0, \\ \frac{\partial \pi_f}{\partial q_f} &= 1 - q_d - 3q_f - t = 0.\end{aligned}$$

The above first order conditions yield the following results:

$$q_d^M = \frac{1}{2}, \quad q_f^M = \frac{1-2t}{6}.$$

Thus, the government's payoff function  $U$  can now be rewritten as

$$U = \frac{19 + 8t - 20t^2 - 12\alpha t(2t - 1)}{72}.$$

Maximizing the function  $U$ , with respect to  $t$ , we get

$$\frac{\partial U}{\partial t} = \frac{2 - 10t - 3\alpha(4t - 1)}{18} = 0,$$

which gives

$$t^M = \frac{3\alpha + 2}{2(6\alpha + 5)}. \quad (1)$$

By using (1), we get the following output level at equilibrium:

$$q_f^M = \frac{\alpha + 1}{2(6\alpha + 5)} \quad (2)$$

Furthermore, the aggregate quantity  $Q^M$  in the market, the firms' profits, consumer surplus, social welfare, tariff revenue and the government's payoff are given by:

$$\begin{aligned}Q^M &= \frac{7\alpha + 6}{2(6\alpha + 5)}, \\ \pi_d^M &= \frac{4\alpha + 3}{8(6\alpha + 5)}, \quad \pi_f^M = \frac{3(\alpha + 1)^2}{8(6\alpha + 5)^2}, \\ CS^M &= \frac{(7\alpha + 6)^2}{(6\alpha + 5)^2}, \\ W^M &= \frac{73\alpha^2 + 122\alpha + 51}{8(6\alpha + 5)^2}, \\ R^M &= \frac{(\alpha + 1)(3\alpha + 2)}{4(6\alpha + 5)^2}, \\ U^M &= \frac{\alpha^2 + 14\alpha + 11}{8(6\alpha + 5)}.\end{aligned}$$

**The Privatized Duopoly**

Now, let us consider the case where the public firm is privatized without cost. So, the objective function of the privatized firm  $F_d$  is now its profit

$$\pi_d = (\alpha - q_d - q_f)q_d - \frac{1}{2}d_d^2. \tag{3}$$

Utilizing the same way of calculation as in the previous sections, we get the following result (throughout the paper, we use the notation subscript  $P$  to refer to privatized firm):

$$\begin{aligned} t^P &= \frac{2(8\alpha + 7)}{48\alpha + 41}, \\ q_d^P &= \frac{2(7\alpha + 6)}{48\alpha + 41}, \quad q_f^P = \frac{6\alpha + 5}{48\alpha + 41}, \\ \pi_d^P &= \frac{6(7\alpha + 6)^2}{(48\alpha + 41)^2}, \quad \pi_f^P = \frac{3(6\alpha + 5)^2}{2(48\alpha + 41)^2}, \\ CS^P &= \frac{(20\alpha + 17)^2}{2(48\alpha + 41)^2}, \\ W^P &= \frac{988\alpha^2 + 1688\alpha + 721}{2(48\alpha + 41)^2}, \\ R^P &= \frac{2(6\alpha + 5)(8\alpha + 7)}{(48\alpha + 41)^2}, \\ U^P &= \frac{(4\alpha^2 + 28\alpha + 21)}{2(48\alpha + 41)}. \end{aligned}$$

**Numerical Example**

Here, we present an example that will help us to illustrate the results presented in the paper. Consider a market with the parameter  $\alpha$ , representing the weight of the government’s preference for tariff revenues, equal to 0.5.

Table 1 shows the results for the mixed and private markets.

**Comparative Static Analysis**

Now, we do a comparative static analysis on the results presented above. In the mixed market, the import tariff imposed by the government is increasing in the weight of the government’s preference for tariff revenues, while it is decreasing when the market is served only by private firms:

$$\frac{\partial t^M}{\partial \alpha} = \frac{3}{2(6\alpha + 5)^2} > 0, \quad \frac{\partial t^P}{\partial \alpha} = \frac{16}{(48\alpha + 41)^2} < 0. \tag{4}$$

Production of the foreign private firm is decreasing in the government’s preference for tariff revenues, when it competes with a public firm, whose

TABLE 1 Results for Mixed and Private Markets

Mixed duopoly	Private duopoly
$t^M = 0.438$	$t^P = 0.338$
$q_d^M = 0.500$	$q_d^P = 0.292$
$q_f^M = 0.094$	$q_f^P = 0.123$
$Q^M = 0.594$	$Q^P = 0.415$
$\pi_d^M = 0.078$	$\pi_d^P = 0.128$
$\pi_f^M = 0.013$	$\pi_f^P = 0.023$
$CS^M = 0.176$	$CS^P = 0.086$
$W^M = 0.254$	$W^P = 0.214$
$R^M = 0.021$	$R^P = 0.042$
$U^M = 0.285$	$U^P = 0.277$

production is constant; after privatization, the production of the domestic privatized firm is decreasing and the production of the foreign private firm is increasing; and the overall effect is that the aggregate quantity in the mixed market is decreasing in the government's preference for tariff revenues, and it is increasing after privatization:

$$\begin{aligned} \frac{\partial q_d^M}{\partial \alpha} &= 0, & \frac{\partial q_f^M}{\partial \alpha} &= -\frac{1}{2(6\alpha + 5)^2} < 0, \\ \frac{\partial q_d^P}{\partial \alpha} &= -\frac{2}{(48\alpha + 41)^2} < 0, & \frac{\partial q_f^P}{\partial \alpha} &= \frac{6}{(48\alpha + 41)^2} > 0, \\ \frac{\partial q^M}{\partial \alpha} &= -\frac{1}{2(6\alpha + 5)^2} < 0, & \frac{\partial q^P}{\partial \alpha} &= \frac{4}{(48\alpha + 41)^2} > 0. \end{aligned}$$

Furthermore, in the mixed market, domestic public firm's profit increases in the government's preference for tariff revenues, while foreign private firm's profit decreases; After privatization, domestic privatized firm's profit decreases in the government's preference for tariff revenues, while foreign private firm's profit increases:

$$\begin{aligned} \frac{\partial \pi_d^M}{\partial \alpha} &= \frac{1}{4(6\alpha + 5)^2} > 0, & \frac{\partial \pi_f^M}{\partial \alpha} &= -\frac{3}{4(6\alpha + 5)^3} < 0, \\ \frac{\partial \pi_d^P}{\partial \alpha} &= -\frac{12(7\alpha + 6)}{(48\alpha + 41)^3} < 0, & \frac{\partial \pi_f^P}{\partial \alpha} &= \frac{18(6\alpha + 5)}{(48\alpha + 41)^3} > 0. \end{aligned}$$

Consumer surplus decreases in the government's preference for tariff

revenues, in the mixed competition, and it increases when both firms are private:

$$\frac{\partial CS^M}{\partial \alpha} = -\frac{7\alpha + 6}{4(6\alpha + 5)^3} < 0, \quad \frac{\partial CS^P}{\partial \alpha} = \frac{4(20\alpha + 17)}{(48\alpha + 41)^3} > 0.$$

Now, we will do a comparative static analysis on the social welfare. We conclude that social welfare decreases in the government's preference for tariff revenues, in both, mixed and private, models:

$$\frac{\partial W^M}{\partial \alpha} = -\frac{\alpha + 1}{4(6\alpha + 5)^3} < 0, \quad \frac{\partial W^P}{\partial \alpha} = -\frac{4(\alpha + 1)}{(48\alpha + 41)^3} < 0.$$

### Effects of Privatization

Comparing the payoff function of the government before and after privatization, we get

$$U^M - U^P = \frac{48\alpha^3 + 39\alpha^2 - 38\alpha - 31}{8(6\alpha + 5)(48\alpha + 41)}, \tag{5}$$

which is positive for values of  $\alpha < \alpha_0$  and negative for  $\alpha > \alpha_0$ , where  $\alpha_0 \in (0.89, 0.90)$  and  $48\alpha_0^3 + 39\alpha_0^2 - 38\alpha_0 - 31 = 0$ . Thus, we get the proposition below.

**PROPOSITION 1.** *In the duopoly model presented above, the government privatizes the public firm just if it put a high weight on the tariff revenues.*

Comparing the other equilibrium outputs in both mixed and privatized models, we get the following proposition that summarizes the effects of privatization.

**PROPOSITION 2.** *In the duopoly model presented above,*  
 (a) *Privatization increases the value of the import tariff, the quantity produced by the foreign firm, the profits of both firms and the import tariff revenue;*  
 (b) *Privatization decreases the quantity produced by the domestic firm, the aggregate quantity in the domestic market, the consumer surplus and the social welfare.*

### Conclusions

In this paper, we studied privatization and government preferences for import tariff revenue in an international Cournot model. First, the government chooses the import tariff to maximize a weighted function of social welfare and import tariff revenue. Second, observing the value of

the import tariff, both domestic and foreign firms simultaneous and independently choose quantities. We presented the equilibrium outcomes of the mixed duopoly and of the privatized duopoly.

By doing a comparative static analysis, we showed that as the preference of the government for tariff revenue becomes large, the social welfare decreases in both mixed and privatized market competitions.

Furthermore, we also analysed the effects of privatization, and we concluded that privatization decreases social welfare.

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